Generating Pulse Compression Waveforms Robust to Eclipsing

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Abstract

Pulse compression is a very computationally intensive part of radar signal processing. This paper studies the design tradeoffs that are available when an optimisation algorithm is used to design non-linear frequency modulated pulse compression profiles in conjunction with window functions to control compression sidelobes in order to be resilient to partial eclipsing. The study has revealed waveforms may be found for some applications that use only conventional pulse compression algorithms, but can be eclipsed by up to at least 50% while retaining a useful level of compression performance with satisfactorily low range-time sidelobes and compression peak width.

1 Introduction

In general, as pulsed radars isolate the receiver during the transmitted pulse width, the first part of the return will be eclipsed for close targets. In spite of the eclipsing loss, such a short range target may nevertheless be detected if there is sufficient energy within the later part of the return which falls within the first range cell. The processing of pulse compression waveforms in a matched receiver [1] represents an autocorrelation of the waveform which is characterised by a main lobe and smaller processing sidelobes, known as range sidelobes, either side of the main lobe. The range sidelobes persist over a duration of twice the transmitted pulse width (for uneclipsed pulses), i.e. from the main lobe peak $\pm T$. These range sidelobes are undesirable because they may be sufficiently strong to trigger the detection of additional (false) targets and/or could mask the detection of genuine small targets in close proximity to the larger one. Range sidelobes may be suppressed by applying a suitable weighting function across the transmitted pulse (or alternatively by weighting the compression reference), which introduces a small degree of mis-match between the received waveform and the receiver. Eclipsing results in the loss of part of the modulation and the remaining fragment of the received pulse no longer fully correlates with the matched filter processing. Eclipsing can accentuate the mis-match in the correlation process leading to degradations in the range sidelobes and to the correlation peak and width.

This paper characterises the effects of eclipsing losses on a variety of traditionally used pulse compression waveforms. It further seeks to propose pulse compression waveforms which minimise the detrimental effects of partial eclipsing, particularly the loss of the first part of the modulation waveform resulting from very close range targets. The loss of the leading pulse section is considered important in active radar missile seeker applications which must provide target data with minimal loss of resolution so that range profiling and aim-point selection may be maintained all the way down to target impact. The authors believe that these waveforms are also of great relevance to other radar applications such as automotive radars.

2 Literature Review

Zrnic *et al* consider the effect of eclipsing losses on linear FM *chirp* pulses and note the loss of range resolution, increased Doppler range coupling and increases in sidelobes [2,3]. They also discuss the use of a window function, e.g. a Hamming window, for the reduction of sidelobes and highlight that the use of a window function results in a degree of filter mismatch, which also causes a reduction in the correlation peak, and hence SNR, and a loss in range resolution.

Many attempts to control the sidelobes of partially eclipsed returns are based on the transmission of standard pulse compression waveforms (e.g. linear FM chirp) and on the processing of returns using a mis-matched filter which is optimised for minimal sidelobes. Since the degree of partial eclipsing is predictable for each range cell, the processing for each range cell may be optimised for the fragment of the received pulse anticipated in each particular range cell; in this way, the processing differs from one range cell to the next [4]. Zrnic et al [3] report on the use of an Iterative Reweighted Least Squares (IRLS) algorithm optimised for sidelobe suppression and minimal degradation to the range resolution and demonstrate superior performance than that obtained from a Hamming weighting function. Blunt and Gerlach [5] present a technique for processing sidelobe reduction based on a Minimum Mean Square error (MMSE) formulation in which the processing in each range cell is based on an estimation adapted from the received data. The algorithm has demonstrated improved sensitivity to small targets in the presence of nearby large targets [5].

Lane [4] presents a sidelobe reduction algorithm to account for Doppler and eclipsing effects called the Thresholded Minimum Mean Square error (MMSE-T) technique. This algorithm relies on an estimate of the background noise and the approximate Doppler shift of a target return. Blunt, Gerlach and Mokole have extended the MMSE technique based on an adaptive pulse compression (APC) algorithm to cope with pulse eclipsing [6] which results in an Eclipsed Repair APC (APC-ER) algorithm. The authors claim that the APC-ER algorithm is less computationally demanding than the MMSE-T technique of Lane [4]. Recent work by Henke, McCormick, Blunt and Higgins [7] has applied optimised mis-matched filtering via a least squares and adaptive pulse compression algorithm to any arbitrary FM waveform for the suppression of processing sidelobes.

Whilst a matched filter results in the best peak signal to mean noise ratio, it is clear that mis-matched filtering algorithms achieve far lower processing sidelobes than the matched filter case, even under the conditions of Doppler corruption and pulse eclipsing. The reduction in sidelobes therefore brings about greater sensitivity of small targets in the presence of considerably larger ones. These algorithms, however, tend to come at the expense of the computational load. A matched filter entails a processing load for each range cell of O(N), where N is the number of samples over the duration of the (transmitted) pulse width; N is therefore a measure of the over-sampling ratio. Zhengzheng Li et al [8] compare the processing loads of some of the above algorithms and state that the processing load for each range cell of the RMMSE algorithm is $O(N^3)$ but that the subsequent efficiency implementation described later in [5] reduce this to $O(N^2)$. Such high processing complexity gives rise to grave concerns as to whether the algorithm could run in real-time. Subsequent work has sought to reduce the processing load.

The discourse offered above indicates that considerable research effort has been invested in the development of algorithms to process standard pulse compression waveforms such that processing sidelobes are suppressed, even when returns are Doppler corrupted and/or eclipsed. Adaptive algorithms applying mis-matched filters are successful in suppressing range sidelobes but inevitably degrade the SNR and range resolution and invoke a higher processing burden. Very much less effort seems to have gone into the design of waveforms which are inherently immune to the detrimental effects of eclipsing.

Many of the MMSE algorithms may still be applied to pulse profiles which are designed to be robust to eclipsing, however, using the MMSE algorithms would result in increased processing requirements. This paper addresses the issues of studying what is possible from a waveform that is designed to provide a useful performance over a wide range of eclipsing conditions, but using only simple correlation processing. It is accepted that the performance of the 'generalised' waveforms will not be necessarily as 'perfect' as the results obtained by applying the more processor intensive adaptive algorithms, but for many applications, the performance of the more general pulse structures may be adequate, but with much simplified signal processing.

3 Characterisation of Traditional Pulse Compression Waveforms

Traditional LFM compression waveforms have the property that as the percentage of eclipsing increases, then the proportion of the bandwidth remaining decreases linearly. The result is that the range resolution cannot be any better than the bandwidth fragment that is not eclipsed, leading to a range resolution which degrades smoothly as the pulse is eclipsed.

The window function that is applied on receive, for simple processing systems, will also effectively be truncated as the degree of eclipsing increases. The window function shape is typically low at the extreme edges and peaks in the centre in order to provide good range sidelobe control. The truncation of the window therefore creates an asymmetric window pattern that now does not have the full desired control effect on the range sidelobes, and also has a SNR loss which is nonlinear with the degree of eclipsing.

A representative example where the use of an eclipsingtolerant waveform would be desirable is that of a 'medium' range mode of a car radar, which should obtain detections from objects as close as possible, but is still required to obtain a detection range out to a few hundred metres and therefore pulses with high peak powers are a useful option. Therefore, a system where the centre carrier is 77GHz and a 100ns (0.1µs) pulse has been transmitted is considered. For a design specification, we have chosen a representative scenario where a maximum of 300MHz sweep bandwidth is available for use. A 300MHz sweep notionally achieves a 0.5m range resolution (when the peak is measured at a -6dB level), and therefore the pulse will need to be compressed with a compression ratio of 30. For a car radar system, the spread of target velocities is likely to lie in the modest interval of -50m/s to +100m/s if the radar is on the front of the vehicle; the corresponding spread in Doppler shift is too small to make any noticeable range-walk effect of the pulse compression peak and therefore the ambiguity function for a single pulse is dominated by the range behaviour only.



Figure 1: Cross correlation profile showing effect of 50% leading-edge eclipsing for a Linear FM chirp with a rectangular window function. Green line shows reference uneclipsed LFM with rectangular window.

Figure 1 shows the effect of 50% eclipsing an LFM waveform with a 'default' rectangular window applied; even when the signal is eclipsed, the remaining fragment of the pulse will still be compressed with a reference that is using a rectangular window. The performance at 50% eclipsing has distorted the range-time sidelobe patterns, although the sidelobe control is still reasonable, given that a rectangular window is applied. Note that in the figures, the peak of the eclipsed waveforms

reduce, but so do the level of the sidelobes; the ratio of the peak to sidelobe levels are detailed in the tables. SNR loss is the reduction in SNR that occurs due to the window shape and a key advantage of the rectangular window is that there is no loss in the maximum available SNR from the uneclipsed section due to the windowing process alone (i.e. the loss from eclipsing is considered unavoidable and not included in the tables).

The rectangular pulse achieves the following behaviour with eclipsing (and given the factor of 30 compression ratio):

Eclipsing Level	Peak Sidelobe relative to compression peak	Width of peak 10dB down relative to uneclipsed rectangular window LFM	SNR loss in the remaining pulse fragment
0%	-13.7dB	0%	0dB
25%	-13.3dB	31%	0dB
50%	-13.3dB	110%	0dB

Table 1: Behaviour of LFM with Rectangular Window as a Function of Eclipsing

The width of the compression peak has been measured at 10dB below the compressed peak level and is a more representative width measurement of the peak, given practical detection thresholds, than the more common 6dB measurement level. At a -10dB level, the width of the reference 300MHz sweep rectangular windowed LFM is 4.88ns which corresponds to a range resolution of 0.73m. Thus the 25% eclipse degrades the range resolution by 31% to 0.96m etc.

Figure 2 shows the effect of eclipsing an LFM waveform that has a Hamming window [1] applied. The performance at 50% eclipsing shown in Figure 2 has a severely degraded range resolution, although the sidelobe control may just still be acceptable for some applications. The Hamming window achieves the following behaviour with eclipsing (and given the factor of 30 compression ratio):

Eclipsing	Peak	Width of	SNR loss	
Level	Sidelobe relative to compression peak	peak 10dB down relative to uneclipsed rectangular window LFM	in the remaining pulse fragment	
0%	-32.1dB	81%	1.35dB	
25%	-23.6dB	260%	0.93dB	
50%	-13.4dB	633%	1.35dB	

Table 2: Behaviour of LFM with Hamming Window as a Function of Eclipsing

The sidelobe level with no eclipsing and the Hamming window applied is not as low as may first be expected from a theoretical Hamming window, i.e. -42.8dB; the degraded performance is due to the low compression ratio of 30. It is worth noting that the use of the Hamming window, with a compression ratio of just 30, results in lower first sidelobe

levels in conditions of up to 50% eclipsing to that of the rectangular window 1^{st} sidelobe level.

The SNR loss is a function of the window shape; as the leading and trailing edges become more heavily attenuated, then the SNR loss increases further. Windows often start at a 'low' level and with 25% eclipsing, the 25% removed will often only be a small fraction of the total area under the curve, corresponding to an SNR loss that degrades non-linearly with increasing levels of eclipsing. At 50% eclipsing, exactly half of the window has been removed and as the window is symmetric, the loss due to the eclipsed shape (i.e. not including the 50% of the pulse energy that is also being lost) is returned to being the same as for the uneclipsed window.



Figure 2: Cross correlation profile showing effect of 50% trailing-edge eclipsing for a Linear FM chirp with a Hamming window function.

The window function is governed by the need to reduce the sidelobes and therefore, any optimisation of the window function for linear FM waveforms is driven by the peak sidelobe level and width of the main compression lobe criteria; attempting to minimise the SNR loss will result in degradations of either the sidelobe levels and/or main compression width. To obtain any independent control of all three criteria (sidelobe levels, main compression width and SNR) requires a non-linear frequency sweep.

4 Design of Novel Pulse Compression Waveforms Resilient to Eclipsing

As an alternative to employing traditional frequency sweep profiles and windows, an optimisation process has been employed to identify non-linear frequency sweep and amplitude taper profiles simultaneously. The taper window shape is therefore designed specifically to match with the changes in the rate of the non-linear sweep; the additional flexibility that is gained through simultaneous optimisation of the sweep and window shape allows non-conventional design trade-offs to be explored. The optimisation process has been implemented using an Evolutionary Algorithm to perform the search for the chirp non-linearity and the window shape simultaneously. The Evolutionary Algorithm used is Differential Evolution [9], although many other forms of Evolutionary Algorithm may provide useful performance too. The compression window function and the non-linear sweep profile to be optimised are defined using a vector of 9 parameters in total. The first 7 parameters of the vector define the window shape to be applied on reception in the crosscorrelation process, and parameters 8 and 9 define the frequency sweep profile. Figure 3 shows an illustration of how the first 7 parameters influence the window shape. The first parameter defines the vertical offset or 'pedestal' level of the left side of the window function (G1). The second parameter (G₂) defines the pedestal level for the right-hand trailing edge of the window which is shaped. The remaining 5 parameters, G₃ to G₇, control the shape of the window between the pedestal edges. The shaping is performed by the parameters representing the amplitude of a set of half-sine waves that are then mapped onto the diagonal formed between the left and right pedestal levels at the edges of the window region. The window shape generated is then scaled to ensure that the highest value in the window has a peak of unity.



Figure 3: Use of parameters G_1 to G_7 to control window shape behaviour.

The process of creating the window is described mathematically in (1). The window uses a sum of half-sine terms in order to generate a parametric description of an arbitrary window shape, but with a limit on how fast it can 'turn', based on the highest sine terms in the parameter vector. A half-sine term starts at zero, and ends at zero when the phase reaches π radians. Thus $\sin(0)=0$ and $\sin(\pi)=0$. Therefore, if 5 sine terms, numbered $1 \le n \le 5$, are summed, each with a weight defined by parameter G_{n+2} , the vertical offset of the window function is given by:

$$w^{*}(t) = G_{1} + t(G_{2} - G_{1}) + \sum_{n \in [1,5]} G_{n+2} \sin(n\pi t)$$

$$w(t) = \frac{w^{*}(t)}{\max_{\forall t} \left(w^{*}(t)\right)}$$

$$t = 0..1$$
(1)

Where the parameter t is scaled from the region zero to one to generate the curve, into the region zero to the pulse length to map into the shaped region of the window. Figure 3 illustrates how the different sine terms are mapped onto the diagonal line between the two pedestal points. The definition of the window shape allows for asymmetric windows to be created in general. If symmetry of the window is to be enforced (such as in the case where it is known the eclipsing may be on either the leading or trailing edge), then the restriction of $G_2=G_1$ can be enforced to set the pedestal levels to be symmetric, and in (1), only the odd terms of n=1, 3, 5, ... are used to ensure reflection symmetry of the curve about the centre of the pulse time.

The remaining two parameters are used to control how the frequency varies during the pulse time. For convenience, the frequency sweep is defined using a cubic polynomial over the time of the transmitted pulse defined by *t*. The time denoted here by the parameter $0 \le t \le T$ which corresponds to the time sweep between zero and *T*, where *T* is the pulse duration. The sweep is to be performed over a bandwidth *B* within the time *T*. Two coefficients control the sweep shape: P₁ and P₂ (which correspond to the parameters G8 and G9, respectively, in the waveform description vector) and, in general, will allow for asymmetric sweeps to be formed. In order to provide a chirp that is rotationally symmetric about the pulse centre, the condition P₁=P₂ can be enforced. Equation (2) is then used to determine the frequency profile of the sweep:

$$f(t) = f_0 + \frac{B}{2} \left(a + bt + ct^2 + dt^3 \right)$$
(2)

Where the coefficients *a,b,c* and *d* control the shape of the frequency function and are calculated first by constraining the start and end of the sweep as being f_0 -*B*/2 when t=0 and f_0 +*B*/2 when t=T giving:

$$f(0) = f_0 + \frac{B}{2}(a)$$

$$f(T) = f_0 + \frac{B}{2}(a + bT + cT^2 + dT^3)$$
(3)

The control parameters P_1 and P_2 are then used to control the rate of change of the curve, therefore fixing the derivative of the frequency with respect to *t*:

$$\frac{df}{dt}(t) = f_0 + \frac{B}{2}(b + 2ct + 3dt^2)$$

$$\frac{df}{dt}(0) = f_0 + \frac{B}{2}(b) = P_1$$

$$\frac{df}{dt}(T) = f_0 + \frac{B}{2}(b + 2cT + 3dT^2) = P_2$$
(4)

Thus 4 simultaneous equations can then be solved to find the four curve coefficients a, b, c and d. The frequency profile can then be generated directly from the cubic polynomial. The phase profile that is needed in order to model the chirp is obtained using the relationship that frequency is the rate of change of phase, therefore integrating the polynomial generates the phase profile:

$$f(t) = f_0 + \frac{B}{2} \left(a + bt + ct^2 + dt^3 \right)$$

$$\phi(t) = \int f(t) dt$$

$$\phi(t) = t \left[f_0 + \frac{B}{2} \left(a + \frac{bt}{2} + \frac{ct^2}{3} + \frac{dt^3}{4} \right) \right]$$
(5)

The correlation function is then generated with the 'reference' phase profile first being multiplied element-by-element with

the window function, before the correlation with the 'received' signal (which may have eclipsing effects included). Once the correlation profile of the waveform has been generated, the quality of the solution is assessed so that the optimisation algorithm can identify the best performing solutions for use in the next iterations of the evolutionary process. Three key criteria are used to define the waveform quality:

- 1. Worst relative sidelobe level,
- 2. Central peak width,
- 3. SNR loss.

For each trial waveform design, the optimisation process therefore aggregates together the scores for the three criteria into a single value which is considered to be representative of the overall solution quality. The trade-off between the three criteria can be adjusted by altering the weighting applied to the objectives, with the final weighted sum value being minimised by the optimiser. When different degrees of eclipsing are also being explored, then the set of three criteria are generated independently for each eclipsing fraction of interest and aggregated prior to the assessment of the objectives.

If the waveform is to use either a single pulse, or use a pulse-Doppler waveform which is 'Low PRF' in nature where all targets lie in the first range ambiguity, then only the leading edge of the pulse will be eclipsed and so needs to be considered. The optimisation process can therefore be targeted to identify solutions which are designed specifically to provide good correlation performance to close targets. The optimiser is therefore given the freedom to choose frequency sweep and window functions that are asymmetric.



Figure 4: Optimised Non-Linear FM frequency sweep profile. Sweep profile optimised for eclipsing of leading left-side section only. Red dashed line shows reference LFM sweep.

The optimisation process has been applied to identify a nonlinear sweep profile and a corresponding weighting window profile to give a practical range resolution of 1.4 metres as the worst case when eclipsed by up to 50% on the leading edge. The specification is not achievable using a linear FM sweep, as even with the rectangular window, the best range resolution at 50% eclipsing was over 1.5m (when the central compression peak width is measured at a -10dB level). It is desired to maintain sidelobe levels that are as good as or better than could have been achieved with a rectangular window (i.e. -13.2dB), and an SNR loss of up to 1dB is considered tolerable.

Figure 4 to Figure 7 show the frequency sweep, window function, and cross-correlation profiles for the optimised pulse. The optimised frequency sweep shown in Figure 4 is interesting as it is asymmetric with a slow rate-of-change of frequency in the region where the pulse is most likely to be eclipsed, and with a much faster rate of change at the end of the pulse. Therefore, even when 50% of the pulse is eclipsed, 65% of the total bandwidth remains in the uneclipsed portion of the pulse.



Figure 5: Optimised window function profile. Window function is optimised in conjunction with the non-linear frequency sweep to tolerate eclipsing of left edge only.



Figure 6: Cross correlation profile showing effect of no eclipsing for an optimised Non-Linear FM chirp with a window function optimised to tolerate eclipsing of one edge only.

As the exact degree of eclipsing that will be experienced is unknown, the window function must be as generic as possible for all the eclipsing possibilities up to 50% eclipsing. Figure 5 shows the window profile from the optimisation process and it is asymmetric with the lowest suppression about 70% into the pulse, which is much less likely to suffer eclipsing. The shape of the second half of the window is also interesting in that when eclipsed by 50%, a 'peak' remains which is similar in nature to how a Hamming or Taylor window may look if shrunk to cover just the remaining 50%. The remaining early phase of the window is tapered to allow the sidelobes to be controlled, even when an unknown fraction of the window is eclipsed. Figure 6 and Figure 7 show the compression behaviour with 0% and 50% eclipsing; the optimisation process was run considering 0%, 25% and 50% eclipsing performance simultaneously. The asymmetric optimised pulse achieves the following behaviour with eclipsing:

Eclipsing Level	Peak Sidelobe relative to compression peak	Width of peak 10dB down relative to uneclipsed rectangular window LFM	SNR loss in the remaining pulse fragment	
0%	-18.2dB	29%	1.20dB	
25%	-18.4dB	43%	0.56dB	
50%	-17.2dB	90%	0.31dB	
75%	-11.5dB	326%	0.46dB	

Table	3:	Behaviour	of	Optimised	Non-Symmetrical	Non-
Linear	FN	I and Optin	nised	d Window a	s a Function of Ecli	psing



Figure 7: Cross correlation profile showing effect of 50% eclipsing for an optimised Non-Linear FM chirp with a window function optimised to tolerate eclipsing of one edge only.

5 Conclusions

The survey of the state-of-the-art pulse compression signal processing methods has shown that although excellent target detection and resolution performance can be gained through adaptive processing techniques even when a modulated pulse is eclipsed, the processing levels required are significant.

The studies to investigate the behaviour of a range of nonlinear Frequency Modulated chirps, in conjunction with bespoke designed tapering windows, have revealed that although there are trade-offs to be considered regarding the different aspects of the waveform performance, there may be occasions where very useful pulse compression profiles can be identified. If a pulse profile can be used which has an inherent high degree of tolerance to eclipsing, even when using conventional correlation-based compression processing, then significant simplification of the radar signal processing chain may be made.

The study has revealed that the key design trade-offs are between the levels of the range-time sidelobes, against the width of the central compression peak and also against the loss of signal-to-noise ratio due to heavily tapered window functions. By studying the output of the optimisation process, it can be seen that, in order to provide control over the width of the compression peak, the widest signal bandwidth possible must be preserved, even under eclipsed conditions. The resulting waveforms therefore have the fastest sweep rate in the time period which is least likely to be eclipsed; for waveforms where just the early period will be eclipsed, then the sweep starts slow and the rate of frequency sweep increases with time.

The optimised window shapes attempt to provide windows to control the range-time sidelobes, even though the exact fraction of the pulse that will be eclipsed is unknown in practice. The window functions tend to weight most heavily the sections of the pulse which are least likely to experience eclipsing; the shaping is often 'bell-shaped' in a local region around the section of the pulse which is unlikely to be eclipsed, with then less energy captured from the regions where eclipsing could be significant (and also where the frequency sweep rate is optimised to be quite slow).

Fundamentally, the study has revealed that the trade-offs involved do not produce a 'perfect' waveform which is narrow with negligible loss or sidelobes, however, waveforms can be designed for many less demanding radar applications where the cost and power consumption of the processing hardware are a very significant design driver.

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